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Muon and Electron Number Nonconservation In a V-A Gauge Model

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ABSTRACT

We analyze muon- and electron-lepton number nonconservation in a pure V-A gauge model. The rates for $\mu \to e\gamma$, $\mu \to ee\bar{e}$, and $K_L \to \mu\bar{e}$ are computed for this model. We find that for a reasonable range of neutral heavy lepton mass these rates are in accord with, but not extremely small compared to, present experimental bounds. We comment on the nonorthogonality of ν_e and ν_μ, and interesting features of the L^- decays.

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Some time ago we discussed a six quark model[1] with only left-handed currents. This is a minimal extension of the "standard" four quark Weinberg-Salam SU(2) × U(1) gauge model[2] which allows CP nonconservation to be incorporated. The alternatives are right-handed currents[3] or proliferation of Higgs bosons.[4] Such a model leads to approximate superweak (or microweak[5]) predictions for CP violation. The model also includes a pair of leptons (L^0, L^-), both massive, and coupled to the W's through a left-handed current, in order to cancel anomalies. The L^- can be tentatively identified with the heavy lepton of mass $\simeq 2$ GeV reported at SPEAR[6] and corroborated at DORIS.[7] This model gives the same predications for atomic physics parity violation as the Weinberg-Salam model.

The general form of the leptonic current is

$$J_\mu = \bar{\ell}_c \gamma_\mu (1-\gamma_5) U \ell_n \tag{1}$$

where $\ell_c = (e^-, \mu^-, L^-)$ and $\ell_n = (\nu_1, \nu_2, L_0)$. U is a general unitary matrix. The massless neutrino produced in association with the electron, ν_e, is given by $\sqrt{|U_{11}|^2 + |U_{12}|^2}\, \nu_e = U_{11}\nu_1 + U_{12}\nu_2$; mutatis mutandis for the muon neutrino ν_μ: $\sqrt{|U_{21}|^2 + |U_{22}|^2}\, \nu_\mu = U_{21}\nu_1 + U_{22}\nu_2$. The known limits on hadron-lepton universality, μe universality, and nonorthogonality between ν_e and ν_μ:

$$\langle \nu_e | \nu_\mu \rangle = \frac{-U_{13}^* U_{23}}{\sqrt{|U_{11}|^2 + |U_{12}|^2}\sqrt{|U_{21}|^2 + |U_{22}|^2}} \approx -U_{13}^* U_{23} \tag{2}$$

imply that[3]

$$|U_{13}^* U_{23}| < 0.055 \quad .$$

If $|U_{13}^{*}U_{23}|$ is nonzero, there are several interesting consequences.

$\mu \to e + \gamma$ Decay

The diagram which contributes in leading order to the decay $\mu \to e\gamma$ is shown in Fig. (1). We have calculated this amplitude to be[9]

$$M(\mu \to e+\gamma) = ie \frac{G_F}{\sqrt{2}} \frac{m_\mu}{32\pi^2} \varepsilon U_{23}^{*}U_{13}\bar{e}\sigma_{\alpha\beta}(1+\gamma_5)\mu\varepsilon^{\alpha}q^{\beta} \tag{3}$$

where $\varepsilon = m_{L^0}^2/m_W^2$. The branching ratio of $\mu \to e\gamma$ to $\mu \to e\bar{\nu}_e\nu_\mu$ is then

$$B\ (\mu \to e+\gamma) = \frac{3\alpha}{32\pi} \varepsilon^2 |U_{23}^{*}U_{13}|^2 \quad . \tag{4}$$

If we take $|U_{23}^{*}U_{13}|^2$ to be 0.3×10^{-2} (see below) and $m_W \simeq 60$ GeV, we find

$$m_{L_0} \simeq 12 \text{ GeV} \sim 30 \text{ GeV for } B = 10^{-9} \ .$$

Such a value for B can be tested very soon by the experiment in progress at SIN.[10] The angular distribution of the decay of the polarized muon is given by $(1+\cos\theta)$, where θ is the angle between the direction of the electron momentum and the direction of the muon polarization. This is due to the left-handedness of our weak currents.

$\mu \to ee\bar{e}$:

In the SU(2) × U(1) gauge theory, there are three classes of diagrams contributing to this process: the photon exchange, the Z exchange, and the W^+W^- exchange; the calculation involved is very similar to that of the process $s \to d+\mu+\bar{\mu}$ previously performed.[11] The final result is

$$M(\mu\to ee\bar{e}) = i\,\frac{G_F}{\sqrt{2}}\,\frac{\alpha}{\pi}\,\varepsilon\,\ell n\,\varepsilon\,U^*_{13}U_{23}\bar{e}\gamma_\beta\left(\frac{1-\gamma_5}{2}\right)\mu\bar{e}\gamma^\beta e \tag{5}$$

to leading order in $\ell n\,\varepsilon$. From this we calculate a branching ratio

$$\frac{\Gamma(\mu\to ee\bar{e})}{\Gamma(\mu\to e\bar{\nu}_e\nu_\mu)} = \frac{3\alpha^2}{16\pi^2}\,\varepsilon^2\log^2\varepsilon\,|U^*_{13}U_{23}|^2 \tag{6}$$

For $m_{L^0} = 10$ GeV, $m_W = 60$ GeV, and $|U^*_{13}U_{23}| \simeq 0.055$, this branching ratio is equal to 0.3×10^{-10}, safely smaller than the experimental limit 6×10^{-9}. It is interesting to observe that although this result depends sensitively on the parameter of the theory, the ratio

$$\frac{\Gamma(\mu\to ee\bar{e})}{\Gamma(\mu\to e\gamma)} = \frac{2\alpha}{\pi}\log^2\varepsilon \tag{7}$$

varies only slowly as one changes m_{L^0}. For the values of m_{L^0} and m_W given, this ratio is equal to 0.06, somewhat larger than the level $\sim(\alpha/\pi)$ which one might, a priori, expect. The reason for this is the $\log\frac{1}{\varepsilon}$ term in the Z-exchange contribution to $\mu\to ee\bar{e}$.

Decay of L^-

If the neutral lepton L^0 associated with the charged lepton L^- of mass 2 GeV is indeed as massive as 10 GeV there are several unusual effects. Decays of L^- such as $e^-\bar{\nu}_e L_0$ are forbidden. We have (summing over neutrino and antineutrino species)

$$\Gamma(L^- \to e\nu\bar{\nu}) \simeq \frac{G_F^{\,2}}{192\pi^3}(m_{L^-})^5(|U_{31}|^2+|U_{32}|^2)$$
$$= \Gamma(\mu\to e+\nu+\bar{\nu})\left(\frac{m_{L^-}}{m_\mu}\right)^5(|U_{31}|^2+|U_{32}|^2). \tag{8}$$

This rate is suppressed considerably by the smallness of the mixing angles; taking $(|U_{31}|^2+|U_{32}|^2) \simeq 10^{-2}$, we find $\tau(L^-\to e\nu\bar{\nu}) \sim 10^{-10}$ sec.

The decays $L^- \to e^-\gamma$ and $\mu^-\gamma$ are expected.[12] We have

$$\frac{\Gamma(L^-\to e\gamma)}{\Gamma(\mu\to e\gamma)} = \left(\frac{m_{L^-}}{m_\mu}\right)^5\left|\frac{U_{13}U_{33}^*}{U_{13}U_{23}^*}\right|^2 \simeq \left(\frac{m_{L^-}}{m_\mu}\right)^5|U_{23}|^{-2}\ . \tag{9}$$

Combining Eqs. (8) and (9), we deduce that

$$\frac{\Gamma(L^-\to e\gamma)}{\Gamma(L^-\to e\nu\bar{\nu})} = \frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu\bar{\nu})}\frac{1}{|U_{23}|^2(|U_{31}|^2+|U_{32}|^2)} \approx 10^{-5} \tag{10}$$

if $\Gamma(\mu\to e\gamma)/\Gamma(\mu\to e\nu\bar{\nu})$ is about 10^{-9}, and $|U_{31}| \approx |U_{32}|$.

Neutrino Reactions

We predict a non-zero coupling of the muon neutrino to e^- and L^-. For sufficiently high incident neutrino energies where the mass differences may be neglected, we get

$$\sigma(\nu_\mu N\to\mu^-X) : \sigma(\nu_\mu N\to e^-X) : \sigma(\nu_\mu N\to L^-X)$$
$$= (1-|U_{23}|^2)^2 : |U_{23}U_{13}^*|^2 : |U_{23}U_{33}^*|^2\ . \tag{11}$$

The second reaction gives the upper bound for $|U_{23}U_{13}^*|^2$ which we estimate[8] as no bigger than 10^{-2}. The third reaction is very interesting, because the L^- tracks may be observable in bubble chamber experiments. This model does not give rise to a large high y anomaly in the reaction $\bar{\nu}_\mu N \to \mu^+X$.

In the version of the model presently discussed, there is no neutrino oscillation. However, it is possible to endow ν_1 and ν_2 with finite, nondegenerate masses, in the model; in that case, there will be neutrino oscillations, as discussed in Ref. 8.

Other Phenomena

There are several classical effects[13] discussed in the literature associated with muon number nonconservation, such as $\mu^- N \to e^- N$ and $\mu\bar{e} \to e\bar{\mu}$, but these effects are too small to have a chance for detection in this model. The decays $K_L \to \mu\bar{e}$ (or $e\bar{\mu}$), or $K_L \to \pi\, e\, \bar{\mu}$ are also difficult to detect; for the former, we have[11] in the free quark approximation

$$M(K_L\to\mu\bar{e}) \sim \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \left(\frac{m_c}{38\mathrm{GeV}}\right)^2 \sin\theta_c \cos\theta_c\, U^*_{13}U_{23}\, f_K\left[\bar{\mu}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)e\right]p_\mu \tag{12}$$

where f_K is the kaon decay constant and p^μ is the kaon four-momentum. This leads to the prediction in this model that

$$\frac{\Gamma(K_L\to\mu\bar{e})}{\Gamma(K_L\to\mu\bar{\mu})} \simeq |U_{13}U^*_{23}|^2 \lesssim 10^{-2} . \tag{13}$$

Experimentally, $BR(K_L\to\mu\bar{e}) < 2.0\times10^{-9}$, a bound five times lower than the one on $BR(K_L\to\mu\bar{\mu})$.

NOTE ADDED: After the submission of this work, we received preprints by S. Glashow and H. Fritzsch on matters.

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REFERENCES

[1] M. Kobayashi and T. Maskawa, Progr. Theor. Phys. 49, 652(1973); H. Harari, Phys. Lett. 57B, 265 (1975). S. Pakvasa and H. Sugawara, Phys. Rev. D14, 305 (1976). J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B (in press). L. Maiani, Phys. Lett. 62B, 183 (1976).

[2] S. Weinberg, Phys. Rev. Letters 19, 1264 (1967); A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8), ed. by N. Svartholm (Almquist and Wiksell, Stockholm, 1968) p. 367. S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970).

[3] R.N. Mohapatra, Phys. Rev. D6, 2023 (1972); R.N. Mohapatra, J. Pati and L. Wolfenstein, Phys. Rev. D11, 3319 (1975).

[4] S. Weinberg, Phys. Rev. Lett. 37, 657 (1976); T.D. Lee, Phys. Rev. D8, 1226 (1973).

[5] B.W. Lee, Fermilab preprint FERMILAB-Pub-76/101-THY.

[6] M. Perl, et al., Phys. Rev. Lett, 35, 1489 (1975); G. Feldman, et al., Phys. Rev. Lett. 38, 117 (1977).

[7] R. Felst, talk given at the Chicago APS Meeting, February, 1977.

[8] B. Pontecorvo, Soviet Physics JETP 26, 984 (1968); S. Eliezer and D. Ross, Phys. Rev. D10, 3088 (1974); S. Eliezer and A. Swift Nucl. Phys. B105, 45 (1976); H. Fritzsch and P. Minkowski, Phys. Lett. 62B, 72 (1976).

A. Mann and H. Primakoff, Penn preprint; D. Bailin and N. Dombey, Phys. Lett. 64B, 304 (1976); E. Bellotti, et al., Lett. Nuovo Cimento 17, 553 (1976).The last paper cited

is used to obtain the bound on $|U_{13}^{*}U_{23}|$. Our detailed analysis on this point will be presented in a forthcoming paper by B. W. Lee and R. E. Shrock, to be published.

[9] During the completion of this work, we received preprints bearing on similar matters from T. P. Cheng and L. F. Li (Phys. Rev. Lett., to be published); W. Marciano and A. Sanda, Rockefeller preprint; J. D. Bjorken and S. Weinberg, SLAC preprint, F. Wilczek and A. Zee, Princeton preprint; S. B. Treiman, F. Wilczek, and A. Zee, Princeton preprint; W. K. Tung, IIT preprint; V. Barger and D. Nanopoulos, Wisconsin preprint.

[10] SIN Physics Report No. 1 (December, 1976) described an experiment in progress by W. Dey, et al.

[11] M.K. Gaillard and B.W. Lee, Phys. Rev. D10, 389 (1974); M.K. Gaillard, B.W. Lee and R.E. Shrock, Phys. Rev. D13, 2674 (1976).

[12] For related discussions see ,e.g., A. Zee and F. Wilczek, Nucl.Phys B78, 461(1976); K. Fujikawa and N. Kawamoto, Phys. Rev. Lett. 35,1560 (1975).

[13] See, e.g., G. Feinberg, Phys. Rev. 110, 1482 (1958); G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Lett. 3, 527 (1959); H. Primakoff and S. Rosen, Phys. Rev. 184, 1925 (1969); ibid., D5, 1784 (1972); B. Pontecorvo, (Ref. 8).

FIGURE CAPTION

Fig. 1 One loop diagram contribution to $\mu \to e + \gamma$ via L_0 .

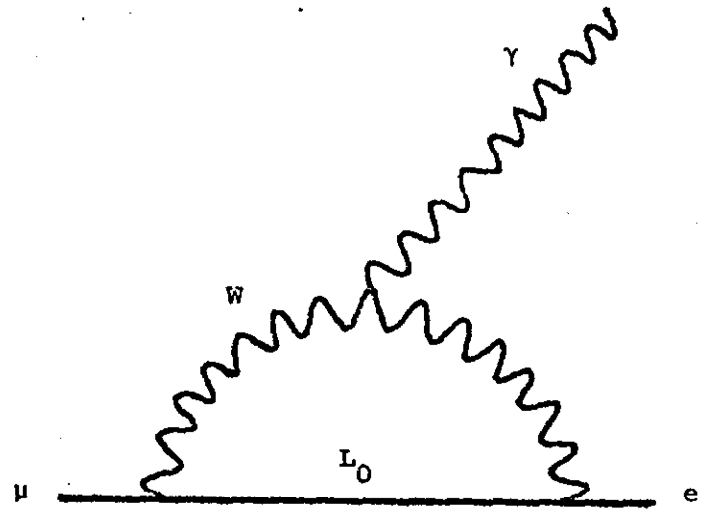

Fig. 1